Let $F(r,t)$ denote some fluid Property of the element of $F(r+dr, t+dt)$ denote the fluid Property at the Point Q.

The rate of change of the fluid element is given as

$$\frac{DF}{Dt} = \lim_{\delta t \to 0} \frac{\delta F}{\delta t} = \lim_{\delta t \to 0} \frac{F(r+\delta r,\ t+\delta t) - F(r,t)}{\delta t}$$

By Taylor's Expansion

$$F(r+q\,\delta t, t+\delta t) = F(r,t) + \left(\frac{\partial F}{\partial t}\right)\delta t + \frac{\partial^2 F}{\partial t^2}(\delta t)^2 + \cdots$$

$$+ \frac{\partial F}{\partial s}(q\,\delta t) + \frac{\partial^2 F}{\partial s^2}(q\,\delta t)^2 + \cdots$$

$$\frac{DF}{Dt} = \lim_{\delta t \to 0} \frac{\partial F}{\partial t} + \frac{\partial F}{\partial s} q$$

This can we write

$$\boxed{\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \frac{\partial}{\partial s} q}$$

Dt = Material derivative

∂t = Local derivative

∂s = Convective derivative

This can be expressed are

(1) As a change due to Local Variation with time of the fluid Property at a given Position.

(2) due to a change of Position at a Prescrived time

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = 0 \quad (1)$$

Where the velocity component u, v, w are Satisfies the continuity eq^n $\nabla \cdot q = 0$

$$\frac{\partial F}{\partial t} = \frac{2x^2}{a^2}\tan t \sec^2 t - \frac{2y^2}{b^2}\cot t \operatorname{cosec}^2 t$$

And $$\frac{\partial F}{\partial x} = \frac{2x}{a^2}\tan^2 t$$

$$\frac{\partial F}{\partial y} = \frac{2y}{b^2}\cot^2 t$$

From (1) we have

$$\frac{2x^2}{a^2}\tan t \sec^2 t - \frac{2y^2}{b^2}\cot t \operatorname{cosec}^2 t + u\frac{2x}{a^2}\tan^2 t + v\frac{2y}{b^2}\cot^2 t = 0$$

$$\Rightarrow \frac{2x}{a^2}\tan t\,(x\sec^2 t + u\tan t) + \frac{2y}{b^2}\cot t\,(-y\operatorname{cosec}^2 t + v\cot t) = 0$$

The Boundary Condition. if

$$x\sec^2 t + u\tan t = 0 \quad (2)$$

And $$-y\operatorname{cosec}^2 t + v\cot t = 0 \quad (3)$$

From (2) And (3)

$$u = -x\sec^2 t\cot t$$

$$u = -(x/\sin t\cos t)$$

and $$v = (y/\sin t\cos t)$$

which satisfies the equation of continuity

$$\nabla \cdot q = 0$$

u, and v with velocity components

Also Normal velocity

$$= \frac{u(\partial F/\partial x) + v(\partial F/\partial y)}{\sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}}$$

or $\frac{\partial^2 v}{\partial t^2} + \frac{\partial}{\partial r}\left\{ v\frac{\partial v}{\partial t} - K\frac{\partial v}{\partial r} + \left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right)v\right\} = 0$

or $\frac{\partial^2 v}{\partial t^2} + \frac{\partial}{\partial r}\left(2v\frac{\partial v}{\partial t} + v^2\frac{\partial v}{\partial r} - K\frac{\partial v}{\partial r}\right) = 0$

or

$$\boxed{\frac{\partial^2 v}{\partial t^2} + \frac{\partial}{\partial r}\left(2v\frac{\partial v}{\partial t} + v^2\frac{\partial v}{\partial r}\right) = K\frac{\partial^2 v}{\partial r^2}}$$

P.d.

Ex-16/99 External Force $= \mu r^{-3/2}$

~~initially~~,

Let v' with velocity at distance r', region at instant of time t.

Let P be the Pressure tube.

Equation of continuity

$$r'^2 v' = f(t) = r^2 v \quad \text{--- (1)}$$

$$\frac{\partial v'}{\partial t} + v'\frac{\partial v'}{\partial r'} = -\mu r'^{-3/2} - \frac{1}{\rho}\frac{\partial P}{\partial r'}$$

$$r'^2 v' = f(t)$$

$$\frac{\partial v'}{\partial t} = \frac{f'(t)}{r'^2}$$

or $\frac{f'(t)}{r'^2} + v'\frac{\partial v'}{\partial r'} = -\mu r^{-3/2} - \frac{1}{\rho}\frac{\partial P}{\partial r'}$

Integrating with regard to r', we have

$$-\frac{f'(t)}{r'} + \frac{1}{2}v'^2 = \frac{2\mu}{r'^{1/2}} - \frac{P}{\rho} + A \quad \text{--- (2)}$$

Initially $r' = \infty$, $v' = 0$, $P = 0$, $A = 0$

$\Rightarrow$ ~~$A = 0$~~ Eq^n (2) Reduces

$$-\frac{f'(t)}{r'} + \frac{1}{2}v'^2 = \frac{2\mu}{r'^{1/2}} - \frac{P}{\rho} \quad \text{--- (3)}$$

$$r^3 v^2 = -\frac{8\mu}{5} r^{5/2} + \frac{8\mu}{5} c^{5/2}$$

$$= \frac{8\mu}{5} (c^{5/2} - r^{5/2})$$

$$v^2 = \frac{8\mu}{5} \frac{(c^{5/2} - r^{5/2})}{r^3}$$

$$v = \frac{dr}{dt} = \pm \sqrt{\frac{8\mu}{5}} \cdot \sqrt{\frac{(c^{5/2} - r^{5/2})}{r^3}}$$

$$\int_0^T dt = \pm \frac{5}{8\mu} \int \frac{r^{3/2}}{(c^{5/2} - r^{5/2})} dr$$

-ve sign is taken between time increases distance r decreases.

Let $r^{5/2} = c^{5/2} \sin^2\theta$

$$\frac{5}{2} r^{3/2} dr = 2c^{5/2} \sin\theta \cos\theta \, d\theta$$

or $$t = \frac{4}{5}\sqrt{\left(\frac{5}{8\mu}\right)} \int_0^{\pi/2} \frac{c^{5/2} \sin\theta \cos\theta}{c^{5/4} \cos\theta} d\theta$$

or $$t = \frac{4}{5} c^{5/4} \sqrt{\left(\frac{5}{8\mu}\right)} \int_0^{\pi/2} \sin\theta \, d\theta$$

$$t = \sqrt{\left(\frac{2}{5\mu}\right)} c^{5/4} \quad \text{Ans.}$$

— x —

Example-1
P-75

$$u(x,y) = \frac{A(x^2-y^2)}{(x^2+y^2)^2}$$

$$v(x,y) = \frac{2Axy}{(x^2+y^2)^2}$$

$$w = 0$$

since there is no external force have

$$X = Y = Z = 0$$

$$w = 0$$

$$\Rightarrow -\frac{1}{\rho}\frac{\partial P}{\partial z} = 0$$

$$\Rightarrow \quad P = P(x,y) \text{ Independent of } z$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$= -\frac{1}{\rho}\frac{\partial P}{\partial z}$$

$$\frac{\partial u}{\partial x} = A\frac{2x(x^2+y^2)^2 - (x^2-y^2)\cdot 2(x^2+y^2)2x}{(x^2+y^2)^4}$$

$$= A\frac{2x(x^2+y^2) - (x^2-y^2)4x}{(x^2+y^2)^3}$$

$$= 2Ax\frac{(3y^2-x^2)}{(x^2+y^2)^3}$$

Similarly

$$\frac{\partial u}{\partial y} = -2Ay\frac{(3x^2-y^2)}{(x^2+y^2)^3}$$

$$\frac{\partial v}{\partial x} = 2Ay\frac{(y^2-3x^2)}{(x^2+y^2)^3}$$

$$\frac{\partial v}{\partial y} = 2Ax\frac{(x^2-3y^2)}{(x^2+y^2)^3}$$